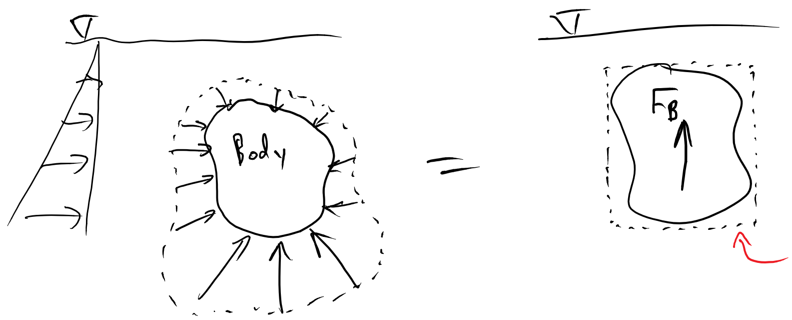
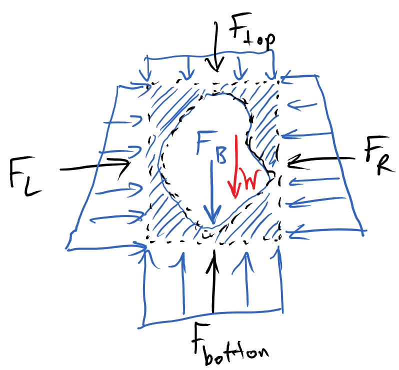
Bouyancy (Archimedes Principle)



Find F_B
"Buoyant force"

control volume that contains the body



F_B is the force of the body on the shaded water

W is the weight of the shaded water

$$\Sigma F_x = 0 \Rightarrow F_L = F_R \Rightarrow \text{They cancel out!}$$

Bouyant force

$$\Sigma F_y = 0 \Rightarrow F_{bottom} - F_{top} - F_B - W = 0$$

$$F_B = (F_{bottom} - F_{top}) - W$$

$$= (P_{bottom} \cdot A_{bottom} - P_{top} \cdot A_{top}) - W$$

(let $A_{bottom} = A_{top} = A$)

$$= (\gamma \cdot z_{bottom} - \gamma \cdot z_{top}) \cdot A - W$$

$$= \gamma \cdot (z_{bottom} - z_{top}) \cdot A - W$$

$$= \cancel{\gamma \cdot h \cdot A} - \gamma \cdot (\cancel{h \cdot A} - V_{body})$$

$h \cdot A$ = volume of the control volume

V_{body} = volume of the immersed body

$h \cdot A - V_{body}$ = volume of water surrounding the immersed body within

$h \cdot A - V_{body}$ = volume of water surrounding the immersed body within the control volume

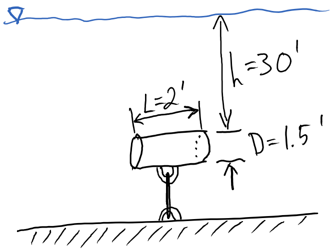
$\ldots \boxed{F_B = \gamma \cdot V_{body}}$

specific weight of the fluid

V_{body} is the volume of the body that is immersed below the free surface

F_B acts at the geometric centroid of the immersed volume

A hollow cylindrical tank weighs $W = 40\,lb$ and is tethered to the bottom of the reservoir, $h = 30'$ below the surface.

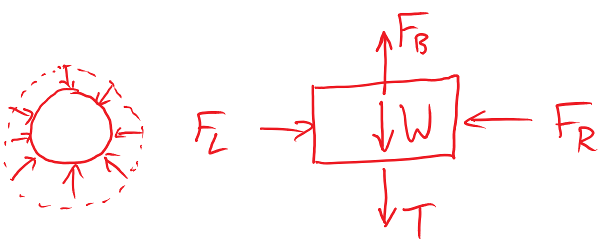


A) Find the tension T in the cable

B) Find the hydrostatic force on the circular face(s) of the cylinder.

FBD

$W = 40\,lbs$



$\Sigma F_y = 0 \Rightarrow F_B = T + W$

$T = F_B - W$

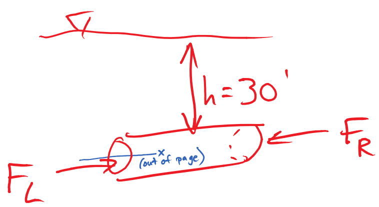
$F_B = \gamma_{H_2O} \cdot V_{body}$

$= \gamma_{H_2O} \cdot \left(L \cdot \frac{\pi}{4} D^2\right)$

$T = \left(62.4 \frac{lb}{ft^3}\right)(2')\frac{\pi}{4}(1.5')^2 - (40\,lb)$

$\boxed{T = 220.5\,lb - 40\,lb = 180.5\,lb}$

B)



$$\begin{cases} F_L = F_R = (p_o + \gamma \cdot h_c) A_{circle} \\ y_p = y_c + \dfrac{I_x}{y_c \cdot A_{circle}} \end{cases}$$

$p_o = 0$

$\gamma = 62.4\, lb/ft^3$

γh_c = pressure at centroid of the circle $= \gamma \cdot \left(h + \frac{D}{2}\right)$

$A_{circle} = \frac{\pi}{4} D^2$

$Y_c = h_c$ = depth of centroid

$I_x = \frac{\pi}{64} D^4$ $\quad h_c = h + \frac{D}{2}$

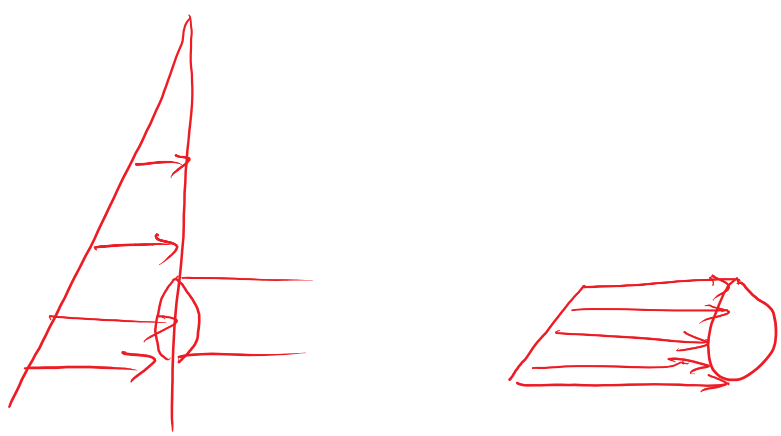
$$\boxed{F_L = \left[0 + \left(62.4 \tfrac{lb}{ft^3}\right)\left(30' + \tfrac{1.5'}{2}\right)\right]\frac{\pi}{4}(1.5')^2 = 3390 \text{ lb}}$$

Location where F_L & F_R act (depth).

$$Y_p = h_c + \frac{I_x}{h_c \cdot A_{circle}} = (30.75') + \frac{\frac{\pi}{64}(1.5')^4}{(30.75')\frac{\pi}{4}(1.5')^2}$$

$$Y_p = 30.75457\ldots \text{ ft}$$

$$\boxed{Y_p \approx 30.75'}$$



Suppose $h = 3'$ and not $30'$

$$h_c = h + \frac{D}{2} = 3.75'$$

$$F_R = \left(62.4 \tfrac{lb}{ft^3}\right)(3.75')(1.767 \text{ ft}^2)$$

$$F_R = 413.5 \text{ lb}$$

$$y_p = 3.75' + \frac{0.2485\ ft^4}{(3.75')(1.767\ ft^2)} = 3.788'$$

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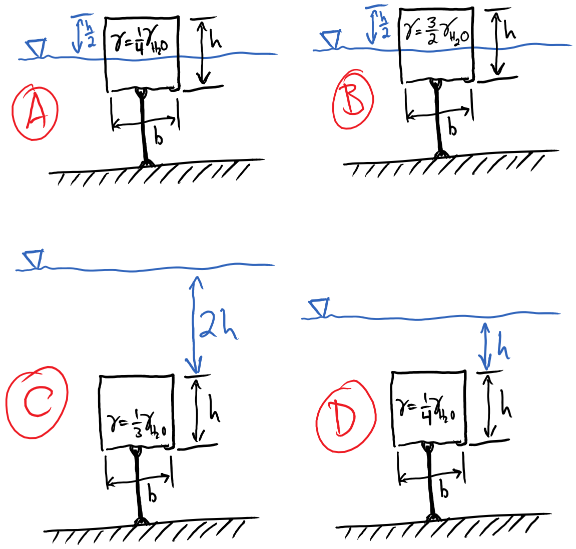
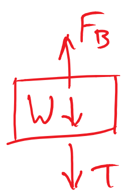
For which case is the tension in the tether greatest?



γ is spec. weight of the box

All boxes have same volume (same width into page)



$$\Sigma F_y = 0$$

$$T = F_B - W$$

$$= \gamma_{H_2O} \cdot V_{immersed} - \gamma_{body} \cdot V_{body}$$

$$= \gamma_{H_2O} \cdot \left(V_{imm.} - S \cdot V_{body}\right)$$

constant e.g. $\gamma = \frac{1}{4}\gamma_{H_2O}$, $S = 1/4$

	T
A	$\gamma_{H_2O} \cdot V_{body}\left(\frac{1}{2} - \frac{1}{4}\right) = \ldots \left(\frac{1}{4}\right)$
B	$\gamma_{H_2O} \cdot V_{body} \cdot \left(\frac{1}{2} - \frac{3}{2}\right) = \ldots (-1)$ ← No tension
C	$\gamma_{H_2O} V_{body} \left(1 - 1/3\right) = \ldots (2/3)$
D	$\gamma_{H_2O} V_{body} \left(1 - \frac{1}{4}\right) = \ldots (3/4)$

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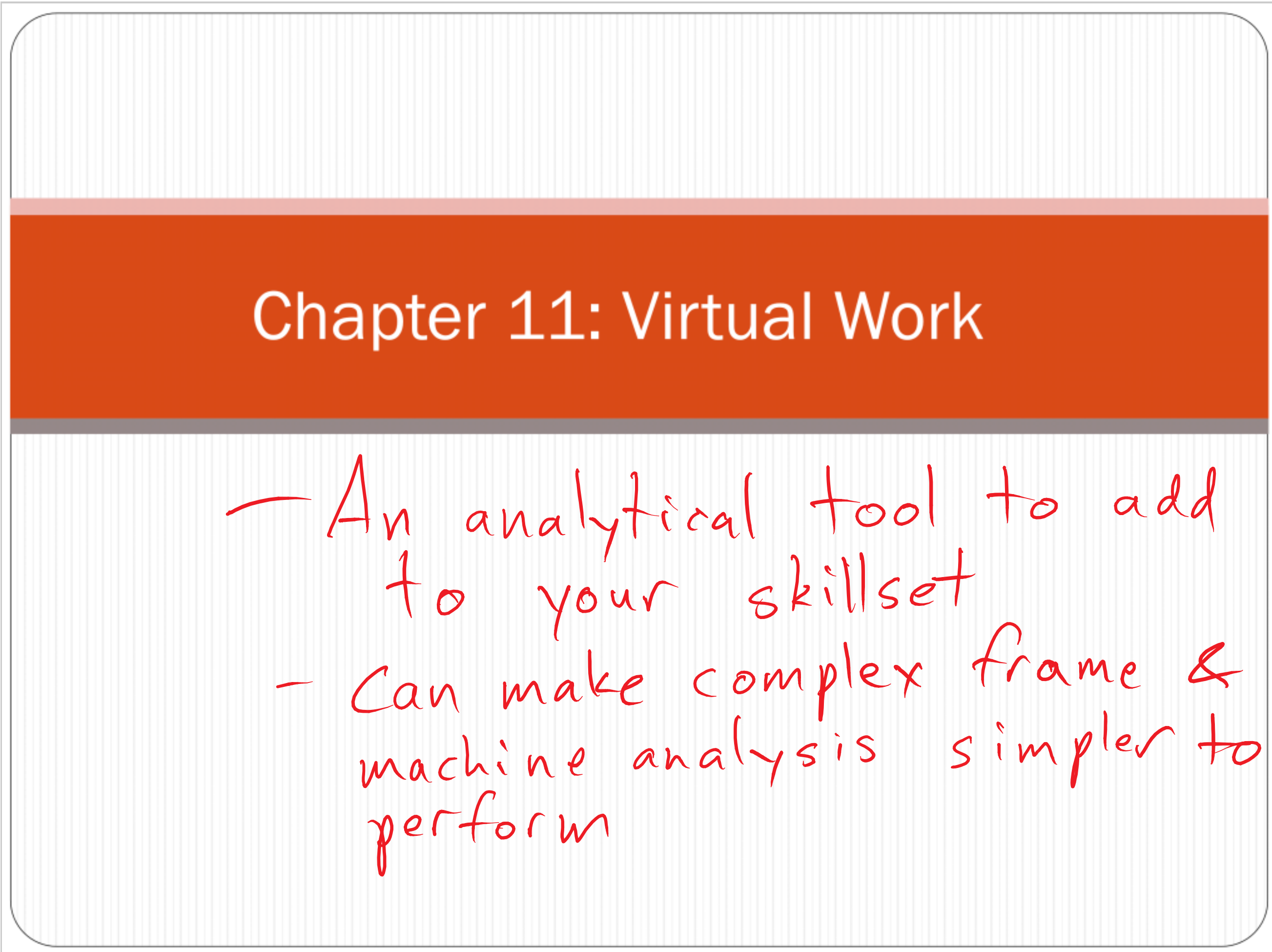

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Dot Products

$\underset{\sim}{a} \cdot \underset{\sim}{b} =$ ____

$$\left\{ \begin{array}{ll} A & |\underset{\sim}{a}| \cdot |\underset{\sim}{b}| \\ B & -|\underset{\sim}{a}| \cdot |\underset{\sim}{b}| \\ C & 0 \end{array} \right.$$

D else

If $\underset{\sim}{a} = (1,5,3)$

$\underset{\sim}{a} \cdot \underset{\sim}{b} =$ $\left\{ \begin{array}{ll} A & 13 \\ B & 11 \end{array} \right.$

$\underset{\sim}{a}$ & $\underset{\sim}{b} = (0, 2, 1)$ $\underset{\sim}{a}$ $\underset{\sim}{b}$ C else

D O

$\underset{\sim}{a} \cdot \underset{\sim}{b} = 1 \times 0 + 5 \times 2 + 3 \times 1$

$= 0 + 10 + 3 = 13$

$\underset{\sim}{a} \cdot \underset{\sim}{b}$ is a $\begin{cases} \text{A. scalar} \\ \text{B. vector} \\ \text{C. tensor} \end{cases}$